The Theory of Computation

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Addison-Wesley-Longman, 1998

Detailed Solution
Chapter 2, Problem 2.34

Another way to phrase this exercise is to say that any undirected graph with at least two vertices has at least a pair of vertices of the same degree—because we can represent people with vertices and the relationship "i knows j" by an edge $\{i, j\}$.

A graph on n vertices has a choice of degrees for its vertices from a minimum of 0 to a maximum of $n - 1$; thus, if no two vertices have the same degree, then the graph must have exactly one vertex of each degree. But that is impossible: since one vertex has degree $n - 1$, it is connected to all other vertices, so that there cannot be a vertex of degree 0. Hence there must be at least one pair of vertices with the same degree.