The Theory of Computation
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Detailed Solution
Chapter 2, Problem 2.44

The hint pretty much solves the problem. Basically, we dovetail on n, the degree of the polynomial, and on $n+1$ copies of the natural numbers to assign the $n+1$ coefficients.

Formally, we proceed by induction. For $n=0$, the set of polynomials of degree 0 is just $\mathbb{Z}$, which is known to be countable. Assume then that the set P_{n-1} of all polynomials of degree not exceeding $n-1$ is countable (our inductive hypothesis) and consider the set P_n of all polynomials of degree not exceeding n. This set is the union of P_{n-1}, a countable set by inductive hypothesis, and of the set of all polynomials of degree equal to n. Thus, by Exercise 2.42, we need only show that the set of polynomials of degree n is countable, since the union of two countable sets is itself countable. Any polynomial of degree n can be written as $a_n x^n + p_{n-1}(x)$, where $p_{n-1}(x)$ is a polynomial of degree not exceeding $n-1$. By inductive hypothesis, we have a procedure to enumerate the latter type of polynomial; the first term is simply a matter of choosing $a_n \in \mathbb{Z}$. Thus we can dovetail on the value of a_n (all integers) and on all polynomials of degree not exceeding $n-1$ and enumerate all polynomials of degree n.

Now that we know that, for each fixed n, the set of all polynomials in one variable of degree not exceeding n is countable, we use one more level of dovetailing, on all positive integers n (degree), to enumerate the set of all polynomials in one variable.