The Theory of Computation

Bernard M.E. Moret

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Detailed Solution

Chapter 4, Problem 6

We need to show that a (deterministic) Turing machine that has three choices for its head movement (stay, move left, or move right) can be simulated by a (deterministic) Turing machine that has only two choices. The simulation is really very simple: we use extra control states and move the head left, then back to the right to simulate the absence of move. Thus, if the augmented Turning machine has a transition of the type

$$\delta(q_i, a) = (q_j, b, -)$$

where the dash denotes that the head stays put, our simulation uses the two transitions

$$\begin{cases} \delta(q_i, a) = ([q_i, a], b, L) \\ \delta([q_i, a], c) = (q_j, c, R) \quad \forall c \in \Sigma \end{cases}$$

while transitions that do move the head are the same in both machines. The notation $[q_i, a]$ denotes a new state for each pair q_i and a such that the augmented Turing machine makes a transition with stationary head when in state q_i reading input a. Our simulation runs in at most twice the number of steps of the original Turing machine, with exactly the same tape contents are all steps; if the Turing machine we are simulating uses an alphabet of k characters and has q control states, then our simulating machine has up to $(k + 1)q$ control states.