The Theory of Computation

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Solution Manual

Chapter 2, Problem 2.11

We are to show that the relation “is big Oh of” is transitive.

If f is $O(g)$, then there exist constants c_1 and N_1 such that, for all $n > N_1$, we have $f(n) \leq c_1 g(n)$; similarly, if g is $O(h)$, then there exist constants c_2 and N_2 such that, for all $n > N_2$, we have $g(n) \leq c_2 h(n)$.

Now simply set $N = \max\{N_1, N_2\}$ and $c = c_1 \cdot c_2$. Then, for all $n > N$, we have $f(n) \leq c_1 g(n) \leq c_1 c_2 h(n) = ch(n)$. Thus we have shown that f is $O(h)$.