The Theory of Computation

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Solution Manual

Chapter 2, Problem 2.19

Together, a graph and its complement (on n vertices) have $n(n-1)/2$ edges. In order for the graph and its complement to be isomorphic, each must have the same number of edges, so that $n(n-1)/2$ must be divisible by 2—or $n(n-1)/4$ must be an integer.

Write $n = 4i + j$; then we must have $(4i + j)(4i + j - 1)/4 = 4i^2 + 2ij - i + \frac{1}{4}(j^2 - j)$ an integer,that is, $j^2 - j$ must be a multiple of 4. Since j can only assume the values 0, 1, 2, or 3, we can check each of the 4 and verify that $j^2 - j = 0$ (a multiple of 4) for $j = 0$ and $j = 1$ and that $j^2 - j$ is not a multiple of 4 for $j = 2$ or $j = 3$.