The Theory of Computation

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Solution Manual

Chapter 2, Problem 2.21

By Exercise 2.20, we have the equality $|R| = |E| - |V| + 2$. A non-trivial region must have at least 3 vertices and 3 edges on its boundary; each edge can be shared by at most two regions. Thus the number of regions is at most 2/3 of the number of edges, $|R| \leq \frac{2}{3}|E|$. Putting the two inequalities together, we get

$$|E| - |V| + 2 \leq \frac{2}{3}|E|$$

or

$$|E| \leq 3|V| - 6$$

as desired.