The Theory of Computation

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Solution Manual

Chapter 3, Problem 3.14

From the hint, it suffices to show that there exists a language that is accepted by a DFA where every state has degree 6 and cannot be accepted by any DFA with at least one state of lower degree. Degree is the sum of in- and out-degrees, so it would be sufficient to demonstrate a DFA with in- and out-degrees of 3 at each state and prove that any DFA with a vertex of lower degree fails to accept that language. (Note that self-loops do not count, nor do two arcs between the same two states.) Degree 6 implies that we must have at least 6 alphabet symbols. Consider the symbols of the alphabet to denote the integer values 1, 2, 3, 4, 5, and 6 and let the language be the set of all strings over that alphabet such that the sum of their digits is one less than a multiple of 7. Note that, from any state of this machine, we must always have 6 distinct outgoing transitions— there cannot be a trap state and using the same transition for two distinct input symbols will irreparably damage our partial sum. Thus the degree of every vertex in the transition state diagram of any DFA for this problem is at least 6 and thus the graph is not planar.

Contrast such a DFA with an NFA for the same problem: the NFA may have vertices of low degree because it may have alternate paths between two states and thus also dead-ends; for instance, we could think of moving from state 3 to state 6 (the one accepting state) in one transition (which needs an input of 3), in two transitions (1 and 2, or 2 and 1, or 4 and 6, or 5 and 5, or 6 and 4), or in three transitions (many choices), using entirely distinct intermediate states (although each such state corresponds to a well-defined partial sum still) and forcing rejection if the second or third transition is not the expected one. The implicit rejection is then the key mechanism in lowering the degree of the nodes.