The Theory of Computation

Bernard M.E. Moret

Addison-Wesley-Longman, 1998

Solution Manual
Chapter 3, Problem 3.18

The point of the problem is to use one or the other of our various closure properties, in order to avoid a difficult application of the pumping lemma (because the language is hard to characterize). Denote our language by L; by definition, L equals $\{0, 1\}^* - \{0^n 1^n \mid n \in \mathbb{N}\}$.

1. *Proof through Closure:* Regular languages are closed under complementation. Thus, if L' were regular, then so would be its complement; but its complement is the language $\{0^n 1^n \mid n \in \mathbb{N}\}$, well known to be nonregular. Hence L' itself cannot be regular. (Other closure-based proofs could be derived, but the one we just gave is clearly the simplest.)
2. *Proof with the Pumping Lemma:* In contrast with a proof based on closure, a proof through the pumping lemma is slightly tricky. Let n be the constant of the pumping lemma and let us pick $z = 0^n 1^{n!+n}$. Clearly, z is in L', since it does not have matching numbers of 0s and 1s. Write $z = uvw$; then v is a string of 0s, so that pumping v a total of i times yields the string $z_i = uv^{i+1}w = 0^{n+i|v|}1^{n!+n}$. Whatever the length of v is (some integer between 1 and n), we can always find an integer i such that $n + i|v| = n! + n$—just pick the value $i = n!/|v|$. The resulting string has matched numbers of 0s and 1s and thus does not belong to L', yielding the desired contradiction and proving that L' is not regular.