The Theory of Computation

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Solution Manual
Chapter 3, Problem 3.19

1. This language is not regular. Let n be the constant of the pumping lemma and choose the string z to be

$$z = \binom{0}{0}^n \binom{1}{1} \binom{0}{0}^n$$

which is clearly in the language. Write $z = uvw$; since $|uv| \leq n$, uv is a string of $\binom{0}{0}$ before the marker $\binom{1}{1}$; thus pumping it up once yields the new string

$$z' = uvvw = \binom{0}{0}^{n+|v|} \binom{1}{1} \binom{0}{0}^n$$

But then the top and bottom rows are both the string $0^{n+|v|}10^n$, which is not its own reverse; hence z' is not in the language.

2. This language is regular: basically, we can only use two of the four symbols of the alphabet, namely $\binom{0}{1}$ and $\binom{1}{0}$. Thus the language has regular expression

$$\left(\binom{0}{1} + \binom{1}{0} \right)^*$$

3. This language is not regular. Let n be the constant of the pumping lemma and pick the string $z = \binom{1}{0}^n \binom{0}{1}^n$, which is clearly in the language. Now write $z = uvw$; v is a string of $\binom{1}{0}$, so that pumping it up just once to $uvvw$ creates more 1s on top than on bottom, so that $uvvw$ is not in the language.