The Theory of Computation

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Solution Manual

Chapter 3, Problem 3.2

We have seen that regular languages are closed under complementation. Thus a pumping lemma for strings not in some given regular language is a pumping lemma for the complement of that language, itself another regular language—so that such a pumping lemma should indeed exist. In fact, we can simply take the normal lemma and exchange throughout "is a member of" and "is not a member of". Let L be our regular language. For strings not in L, we choose such a string z of sufficient length, let the adversary break it into substrings $z = uvw$, and demonstrate the existence of an i such that $z' = uv^i w$ is in the language, thereby showing that the string z, although long enough to force a putative automaton of n states to loop, cannot be pumped and always stay outside the language, which implies that the complement of the language (and thus the language itself) is not regular.

We can use this version of the pumping lemma to prove that a language is not regular by proving directly that its complement is not regular—although using our normal pumping lemma together with our knowledge of the closure of regular languages under complementation has exactly the same effect.