The Theory of Computation
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Solution Manual
Chapter 3, Problem 3.29

The language is regular. Let M be a DFA for L, with starting state q_0, final state set F, and transition function δ. We construct a new machine, M', which recognizes our language. M' basically runs one copy of M forward on x, the other (nondeterministically) backwards on x; if the two copies end in the same state, then the string is accepted.

Formally, we can write a state of M' as a pair $[q_i, q_j]$, where q_i is the state of the copy of M that processes x forwards and q_j is the state of the copy of M that processes x backwards. In addition, M' has an initial state q' from which there are ϵ transitions to each state of the form $[q_0, q_f]$ for $q_f \in F$. The final states of M' are all states of the form $[q_i, q_i]$. If M has transition $\delta(q_i, a) = q_i'$, then M' can move from state $[q_i, q_j]$ to any state $[q_i', q_j']$ such that M has a transition $\delta(q_j', a) = q_j$.