The Theory of Computation

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Addison-Wesley-Longman, 1998

Solution Manual
Chapter 4, Problem 5.11

We can use the addition to keep doubling the value stored in a register and register transfer to copy the result in a collection of registers. Consider the small RAM program below.

```
    R1 <- 4
    R2 <- 1
    R3 <- n       /* other registers are all 0 */
10: R2 <- R2 + R2
    R[R1] <- R2   /* R[R1] is the register indexed by R1 */
    inc R1
    dec R3
    jnz R3 10     /* jump to 10 if R3 is nonzero */
```

This program will store values 2, 4, 8, ... , 2^n in registers 4 through $n + 4$. When started, the program uses $\log n$ bits (in register 3); at completion, however, registers 4 through $n + 4$ use respectively 1, 2, 3, ... , n bits, for a total of $\Theta(n^2)$ bits. The running time is the constant time used in each loop times the number of loops and so is $\Theta(n)$. Thus this small program uses space at a rate that grows quadratically as a function of time.