The Theory of Computation

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Solution Manual
Chapter 4, Problem 5.3

A 2-tape Turing machine can recognize the language of palindromes in linear time with three passes as follows. In the first pass, it copies the input string to its second tape; at the end of this pass, both tapes have the same string on it and both heads are at the end of the input string on both tapes. In the second pass, it moves its first head back to the beginning of the string, leaving its second head immobile; at the end of this pass, the head on the first tape is at the beginning of the input string and the head on the second tape is at the end of the input string. In the third pass, the machine scans both tapes, forward on the first tape and backward on the second tape, moving the two heads together, and verifying that the two symbols read are always equal to each other. Since each pass takes linear time, the entire procedure takes linear time.

On the other hand, a 1-tape Turing machine appears unable to carry out the same task in better than quadratic time: once it has scanned one symbol of the input string, it needs either to fetch the matching symbol (which, for at least one quarter of the symbols, is at least $n/2$ tape squares away, so that fetching just these $n/4$ matching symbols will takes at least $n^2/8$ moves, i.e., quadratic time) or to store the symbol just scanned in a place from which the matching symbol will be easily accessible (which appears to entail exactly the same problem: just moving past the end of the input string to copy the scanned symbol takes linear time per symbol and thus quadratic time overall).