The Theory of Computation

Bernard M.E. Moret

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Solution Manual

Chapter 5, Problem 5.15

1. Define the function $g(<x, y>) = \phi_{\text{univ}}(x, x) - \phi_{\text{univ}}(x, x)$; because this function is partial recursive, there exists some index j such that $\phi_j(<x, y>) = g(x, y)$. By the s-1-1 theorem, there exists an index $k = s(j, x)$ such that $\phi_k(y) = \phi_j(<x, y>)$ for each fixed x. But note that $\phi_k(y) = \phi_j(<x, y>) = g(<x, y>) = \phi_{\text{univ}}(x, x) - \phi_{\text{univ}}(x, x)$ is totally undefined whenever $x \notin K$ and is the constant function ϵ whenever $x \in K$. Thus k is the index of a constant function if and only if $x \in K$; since K is not recursive, neither is the set of constant functions.
2. The very same definition works: the function we obtained is totally undefined if and only if x is not in K, the desired reduction. (A constant function is just a special case of a not totally undefined function.)
3. Define the function $g(<x, y>) = y + \phi_{\text{univ}}(x, x) - \phi_{\text{univ}}(x, x)$; because this function is partial recursive, there exists some index j such that $\phi_j(<x, y>) = g(x, y)$. By the s-1-1 theorem, there exists an index $k = s(j, x)$ such that $\phi_k(y) = \phi_j(<x, y>)$ for each fixed x. But note that $\phi_k(y) = \phi_j(<x, y>) = g(<x, y>) = y + \phi_{\text{univ}}(x, x) - \phi_{\text{univ}}(x, x)$ is totally undefined whenever x is not in K and returns its argument y otherwise. Thus k is not in our set whenever x is not in K and it is in our set (because the range of the function is then the whole of $\mathbb{N}$ and thus encompasses the indices of all total functions and thus of at least one total function) whenever x is in K.