The Theory of Computation
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Solution Manual
Chapter 5, Problem 5.4

Recursion is the key to these definitions. We begin with the existential quantifier; write it as the function $f(x, z_1, \dots, z_n)$. We can easily write the base case as

$$f(0, z_1, \dots, z_n) = P(0, z_1, \dots, z_n)$$

and can then use the step to write

$$f(n+1, z_1, \dots, z_n) = f(n, z_1, \dots, z_n) \text{ or } P(n+1, z_1, \dots, z_n)$$

describing the fact that the function is true (the bounded existential quantifier succeeds) whenever a value at most n is found that works (the recursive call) or the value $n+1$ itself works.

The bounded universal quantifier is the same, except that it uses a logical *and* rather than a logical *or* to join the result of the recursive call and the evaluation of the predicate on the value $n+1$. Writing $g(x, z_1, \dots, z_n)$ for this predicate, we get

$$\begin{cases} g(0, z_1, \dots, z_n) = P(0, z_1, \dots, z_n) \\ g(n+1, z_1, \dots, z_n) = g(n, z_1, \dots, z_n) \text{ and } P(n+1, z_1, \dots, z_n) \end{cases}$$