The Theory of Computation

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Solution Manual

Chapter 5, Problem 5.7

This is actually very simple: all we need is an external induction, much like our definition of the family of concatenation functions. We have $f_0(x) = \text{Succ}(x)$, which is primitive recursive and $f_1(\epsilon, x) = P_1^1(x)$ and $f_1(ya, x) = \text{Succ}(P_2^3(y, f_1(y, x), x))$, which is also primitive recursive. Now let us assume that all $f_i(y, x)$ are primitive recursive up to some $i = k$ and let us prove that $f_{k+1}(y, x)$ is also primitive recursive. By induction, it will follow that all f_i in the family are primitive recursive. We have, by definition, $f_{k+1}(\epsilon, x) = P_1^1(x)$, which obeys all requirements for the basis of primitive recursion, and $f_{k+1}(ya, x) = f_k(P_2^3(y, f_{k+1}(y, x), x))$, which obeys all requirements for the step in primitive recursion. Since all functions used in the definition are primitive recursive (trivially or by inductive hypothesis) and since the definition is a valid application of primitive recursion, the resulting function is primitive recursive.