The Theory of Computation

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Solution Manual
Chapter 5, Additional Problem 5.4

We use the non-repeating enumerator for S, say f_S. We know that the set $K = \{x \mid M_x \text{ halts on } x\}$ is r.e., but non-recursive. Define the new set $S' = f(K)$—i.e., let the set S' be composed of those elements of S that are enumerated in ith position, where i is such that $M_i(i)$ is defined. By construction, S' is a subset of S. It is non-recursive, because K is non-recursive: otherwise, we could, given some element x, decide whether it belongs to K by mapping it through f to an element of S' and deciding membership in S'. Finally, S' is r.e., because it is produced by an enumerator in the first place (enumerate K and feed that through f).