The Theory of Computation

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Solution Manual
Chapter 6, Problem 6.16

The only trick in padding is making sure that the padding need not be handled at all by the existing algorithm—e.g., by adding a distinct separator between the "real" input and the padding. The other consideration is the amount of padding needed, but that is fairly simple: we want to pad an input of size n to size n' so that a running space of $g(f(n))$ appears to be just $g(n')$. In other words, we need to pad enough that the input of size n becomes a new input of size at least $f(n)$; perhaps the simplest way to do this is the make the new input of size $f(n)$ by tacking onto the original input a separator and $f(n) - n$ useless symbols. The algorithm that reads the input need not read beyond the separator, even though the input is much longer; the running space is gauged in terms of the total input size, however, so that the original input of size n that took $g_1(f(n)$ space to process, now appears to take only $g_1(n')$ space to process—even though nothing has changed in the processing. Since the padded problem is solvable within $g_1(n')$ space, it is also solvable within $g_2(n')$ space. The algorithm that runs in g_2 space may not "know" enough to stop reading at the separator, but we can simulate it to ensure that nothing gets read past the separator (a very simple simulation, where we simply check at every move whether the separator has been read and, if so, simulate the transition made by the original machine on end of input tape). This simulation takes the same amount of space as the machine it simulates plus $\log n'$ space to keep track of the original head position on the input tape and thus runs in $g_2(n') + \log n'$ space; in terms of n, this is a running space of $g_2(f(n)) + \log f(n)$ space, which is at most $2 \cdot g_2(f(n))$ space (because we have assumed that the g_is are at least logarithmic). By the compression theorem, we can reduce this quantity to just $g_2(f(n))$ space by a change in alphabet.

Note that this argument requires f to be at least as large as n everywhere, so that $f(n) - n$ is non-negative, and the g_is to be at least logarithmic, so that their contribution is at least as large as that of the counter used to keep track of the input head position.