The Theory of Computation

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Solution Manual

Chapter 6, Problem 6.2

Let an instance of *Partition* be given by a set S and a size assignment $s\colon S \to \mathbb{N}$, such that the sum of all sizes is even, $\sum_{x \in S} s(x) = 2n$ for some $n \in \mathbb{N}$. We shall map the elements of *Partition* into elements of *Kth Largest Subset*, with identical size assignment. We also set $B = n$. We then start a binary search process to determine the exact number of subsets with total size less than or equal to n; since there are up to $2^{|S|}$ subsets, a binary search will take $\log_2 2^{|S|} = |S|$ calls to the routine for *Kth Largest Subset*, each time with a different value of K. What we need to know, however, is the number of distinct subsets with total size *equal* to n: if that number is nonzero, then we can answer "yes" for our original instance of *Partition*. So we now set $B = n - 1$ and conduct the same binary search again; with another $|S|$ calls to the routine for *Kth Largest Subset*, we determine the number of subsets with total size less than or equal to $n - 1$. The difference between our first number and our second one is the desired number of subsets with total size equal to n. If that number is 0, we answer "no," otherwise we answer "yes." The reduction itself does very little work beyond the calls to the subroutine for *Kth Largest Subset*: the mapping of elements and sizes is mere copying of the input and the rest involves only keeping track of two counters of $|S|$ bits each and making one subtraction. The main work is in calling the routine for *Kth Largest Subset* a total of $2|S|$ times.