The Theory of Computation
Bernard M.E. Moret
Addison-Wesley-Longman, 1998

Solution Manual
Chapter 6, Problem 6.23

Savitch's theorem gives us a solution: take the nondeterministic algorithm and make it deterministic through the construction of the theorem. Since that is a rather massive undertaking, we can instead describe its simplified version for the specific problem of *Digraph Reachability*.

If vertex n can be reached from vertex 1, it can be reached through a path with at most $n - 1$ arcs. Our algorithm will use divide-and-conquer, recomputing everything as needed from one part of the problem to the other. In general, we will be looking for a way to reach vertex j from vertex i in at most k steps; we will do so by trying every possible vertex (including i or j) as an intermediate vertex that we can reach from i in at most $k/2$ steps and from which we can reach j in at most $k/2$ steps, solving each subproblem recursively, until one choice of intermediate vertex works (exit current call with success) or no intermediate vertex can be found (exit current call with failure). The depth of recursion is $\log n$ since we halve the path length at each step; the local storage is three vertices (source, destination, current choice of intermediate vertex), which takes logarithmic space. Hence the total storage used is $O(\log^2 n)$, as desired.