The Theory of Computation
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Addison-Wesley-Longman, 1998

Solution Manual
Chapter 6, Problem 6.24

It is easily seen that any polynomial function is subexponential; thus SubExp is closed under polynomial-time transformations, so that it makes sense to use such transformations for reductions inside SubExp. It follows immediately that, if some NP-complete problem is in SubExp, then NP is a subset of SubExp. From the definition of the class, we can immediately write $P \subseteq \text{SubExp} \subseteq \text{Exp}$. Are these containments proper? Consider the family of functions $2^{(log n)^k}$, for $k > 1$; these functions are not polynomial, yet they are subexponential; since

$$\lim_{n\to\infty} \frac{m \cdot \log n \cdot n^m}{2^{(\log n)^k}} = 0,$$

for any constants $k, m > 1$, the hierarchy theorem for time tells us that the two classes P and SubExp are distinct. Since we do not have a general form for the fastest growing functions in SubExp, separating it from Exp is harder, but can also be done on a similar basis: any problem that belongs to SubExp can be solved in time $O(f(n))$, where f is a subexponential function; every subexponential function grows strictly more slowly than any exponential, so that the hierarchy theorem for time tells us that SubExp is a proper subset of Exp. Now SubExp $\subseteq$ NP (or SubExp $\subseteq$ PSpace) would imply $P \neq NP$ ($P \neq$ PSpace), which we do not know. Conversely, the straightforward simulation of NP (and, *a fortiori*, of PSpace), can require true exponential time, so that we do not know whether containment holds the other way either. (All these remarks carry over, of course, to any class in the polynomial hierarchy.) This leaves the relationship between SubExp and L^k and NL^k, for $k > 1$. If a language is in L^k, then it can use at most $c^{((\log n)^k}$ time for some constant c; thus it is in SubExp, by a remark made above; hence $L^k \subseteq$ SubExp, for any k; by Savitch's theorem, it follows that $NL^k \subseteq$ SubExp as well. Thus PolyL $\subseteq$ SubExp. The containments are presumably proper, but our characterization of NL^k does not allow an answer.