The Theory of Computation
Bernard M.E. Moret
Addison-Wesley-Longman, 1998

Solution Manual
Chapter 6, Problem 6.4

The reason is simply that $\mathscr{C}$-completeness requires membership in the class $\mathscr{C}$, whereas $\mathscr{C}$-equivalence does not.