The Theory of Computation

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Solution Manual
Chapter 6, Problem 6.9

Let the problem be Π_0 and denote the larger class by C_2, the smaller class by C_1. Proving Π_0 to be C_2-complete proves that Π_0 belongs to C_2; thus it remains only to show that Π cannot belong to C_1, assuming that the completeness was shown under a reduction that uses resources no larger than those available in C_1. We show that, if Π_0 is in C_1, then we must have $C_1 = C_2$, thereby contradicting the hypothesis of proper containment. Let Π be some arbitrary problem in C_2; since Π_0 is C_2-complete, Π reduces to Π_0, using no more resources than available in C_1. If Π_0 is in C_1, then we can solve with the resources available in C_1; thus, by combining the algorithm for the reduction from Π to Π_0 and the algorithm for the solution of Π_0, both of which run with the resources available in C_1, we get a solution algorithm for Π which runs with the resources available in C_1, leading us to conclude that Π is in C_1. Since Π was an arbitrary problem of C_2, we conclude that C_2 is a subset of C_1, a contradiction.

Note that the only somewhat vague point in this argument is whether or not the reduction used is transitive; but note that it must be transitive in order for the notion of completeness to have any use.