The Theory of Computation

Bernard M.E. Moret

Addison-Wesley-Longman, 1998

Solution Manual

Chapter 6, Additional Problem 6.1

Closure under union and intersection means simply that the union or intersection of two sets in P or NP is also in P or NP, respectively.

Let A and B be two sets in P and let M_A be the solver for set A and M_B the solver for set B. We can combine the two solvers to obtain a solver for $A \cup B$ simply by running each solver in turn on the instance and accepting whenever at least one of the two solvers accepted. Similarly, we can combine the two solvers to obtain a solver for $A \cap B$ simply by running each solver in turn on the instance and accepting whenever both solvers accepted.

Let A and B be two sets in NP and let M_A be the checker for set A and M_B the checker for set B. Every instance in either set has a concise certificate that can be verified in polynomial time. But then every element in $A \cup B$ has such a certificate as well (the same certificate, or one of the two for elements of $A \cap B$), so that the set $A \cup B$ is in NP. In order to be in $A \cap B$, an instance must be in both A and B and thus must possess two concise certificates, one for each set. Thus we can define the set $A \cap B$ by a checker M that runs first M_A on the first part of the certificate and then M_B on the second part of the certificate, accepting only if both parts are verified. Thus $A \cap B$ is in NP.