The Theory of Computation

Bernard M.E. Moret

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Solution Manual
Chapter 8, Problem 8.20

In all three cases, the straightforward multiplication technique works.

1. Given a claimed algorithm $\mathscr{A}$ that guarantees a distance K approximation, we solve an arbitrary instance $S, \mathscr{C} \subseteq 2^S$ of the set cover problem exactly as follows. We produce the new instance $S', \mathscr{C}' \subseteq 2^{S'}$ where S' is the disjoint union of $(K+1)$ copies of S and, similarly, $\mathscr{C}'$ is the disjoint union of $(K+1)$ copies of $\mathscr{C}$. Note that the minimum cover for this new instance is exactly $(K+1)$ times the size of the minimum cover for the original instance. We apply $\mathscr{A}$ to this new instance. Since the cover it returns has at most K subsets more than the minimum cover, $\mathscr{A}$ in fact must have solved one of the copies optimally.
2. Given a claimed algorithm $\mathscr{A}$ that guarantees a distance K approximation, we solve an arbitrary instance $V, \mathscr{C}$ of *2SAT* (where V denotes the set of variables and $\mathscr{C}$ the set of clauses) exactly as follows. We produce the new instance $V', \mathscr{C}'$, where V' is the disjoint union of $(K+1)$ copies of V and $\mathscr{C}'$ is the disjoint union of $(K+1)$ copies of $\mathscr{C}$. Note that the maximum number of clauses satisfiable in this new instance, call it M, is exactly $(K+1)$ times the maximum number satisfiable in the original instance, call it m; that is $M = (K+1)m$. We apply $\mathscr{A}$ to this new instance. Since it returns a truth assignment that satisfies at least $M - K$ of the clauses and since no more than m clauses can be satisfied in each copy, it must have produced a truth assignment that satisfies exactly m clauses in one of the copies.
3. Given a claimed algorithm $\mathscr{A}$ that guarantees a distance K approximation, we solve an arbitrary instance $G = (V, E)$ exactly as follows. We produce a new instance $G' = (V', E')$ which consists simply of $(K+1)$ copies of G. Note that G' will be bipartite if and only if each of the $(K+1)$ copies is bipartite; indeed, an optimal solution for G' has exactly $(K+1)$ times the size of an optimal solution for G. We apply $\mathscr{A}$ to G'; since it returns a subset with at most K vertices more than in the optimal solution, it must have found a minimal subset for at least one of the copies.