The Theory of Computation

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Solution Manual

Chapter 8, Problem 8.23

The key insight in the proof is to realize that the set of edges chosen by the algorithm forms a matching of the graph—i.e., no two edges chosen by the algorithm share a vertex. Since each edge must be covered and since the set of edges examined by the algorithm forms a matching, the minimum vertex cover must include at least one vertex for each such edge; but the cover found by the algorithm includes exactly two vertices for each such edge and thus has at most twice as many vertices as the optimal cover.