The Theory of Computation

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Solution Manual
Chapter 8, Additional Problem 8.1

Denote by $scriptE_i$ the event that the ith clause is not satisfied. Since each clause has exactly k literals, the probability that a given clause is not satisfied is simply $p = 2^{-k}$. Satisfaction of one clause can depend only on other clauses that share a variable and there are at most $k \cdot 2^k/(ke)$ clauses that can share a literal with a given clause; thus we have $d + 1 = 2^k/e$. We can then verify that we have $p(d + 1) \leq 1/e$:

$$p(d+1) = 2^{-k} \cdot 2^k/e = 1/e$$

By Lovasz' local lemma, we thus must have

$$\Pr[\bigcap \overline{\mathcal{E}_i}] > 0$$

which means that the probability that all clauses are simultaneously satisfied by a randomly chosen assignment is nonzero. Hence the instance is satisfiable.